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INTERNAL TIDAL WAVES IN THE OCEAN

By B. Haurwitz

Abstract

The theory of internal tidal waves is briefly reviewed in order to demonstrate the importance of simultaneous observations at different positions. After a description of the expectancy test it is shown that a reasonably high probability exists that the internal tidal oscillations found at Meteor Station 385 in the temperature observations are real and not purely formal results of the harmonic analysis. The existence of internal tides at Meteor Station 438 and, to a lesser degree, at an anchor station of the Altair cannot be regarded as established because of the high probability that amplitudes of the magnitude obtained by harmonic analysis would occur in random data. The study indicates clearly that longer series of observations are required if the existence of internal tidal waves is to be established securely.

1. Introduction

Internal waves in the ocean have received considerable attention in recent years. Apart from the intrinsic interest which these phenomena deserve it can be surmised that they play an appreciable role in furthering mixing processes in those regions of the ocean where they occur. This has become especially apparent after Defant (1948) has made it very probable that they are, at least in the regions of the continental shelfs, subject to breaking, like the waves at the ocean surface in shoaling water.

For the practice of oceanographic surveys it is of course necessary to know if internal waves are present in order to eliminate the effects of such waves on the analysis and interpretation of the data. The importance of this correction has recently been demonstrated very convincingly by Defant (1950).

The effect of waves at the ocean surface on ship motions is well known and does not need any further elaboration. The "dead-water" phenomenon caused by internal waves generated by the motion of ships has also been fully described in text books, for instance by Sverdrup and Coll. (1942). In a similar, but much stronger way submarine navigation may be affected by internal waves at lower depths. And it appears that submarines have encountered such internal waves, with rather unpleasant results.

As far as the excitation of internal waves is concerned it can be surmised that they are in part due to atmospheric causes, like the ocean surface waves. Other internal waves may be caused by seismic activity, but in view of the relatively infrequent occurrence of earthquakes, as compared to the apparently quite

frequent occurrence of internal waves, it is necessary to attribute the origin of at least many of the observed internal waves to other causes.

Since one of the most important forces producing periodic changes in the oceans is the tidal action of the sun and the moon particularly attention has naturally been given to oscillations of tidal periods in the study of internal waves.

In a previous report (Haurwitz, 1952) various papers were listed which present observations of internal tides. Further, a series of temperature soundings made by the Atlantis from 16 to 29 June 1938 at latitude 34°N and 66°W, northwest of Bermuda was analyzed in order to determine the existence or nonexistence of internal tides. A statistical test disclosed that no internal waves of tidal character were present in the Atlantis material, since the amplitudes of the oscillations determined by harmonic analysis are not larger than can be expected in random data. This conclusion suggested further that some of the other reported occurrences of internal tidal waves might not be real, but simply due to the formal harmonic analysis of the observations.

With these considerations in mind some further observation material has since been subjected to the same statistical tests, and the results of this study are being presented here. Specifically, the data considered are those obtained by the German North Atlantic Expedition with the Meteor in 1937 and 1938 (von Schubert, 1944) and by the International Gulfstream Investigation with the Altair 1938 (Defant, 1940a) which have been readily

accessible to the author. The analysis has in every case been confined to the temperature, as in the case of the Atlantis data. For these three series of observations which are treated here salinity and current measurements are also available. But these data were not analyzed since the necessity of statistical tests can be demonstrated quite clearly on the basis of the temperature observations alone. Such tests require a sufficient amount of data. Hence, for reasonably reliable determination of the presence of internal tides and their nature the observations should comprise more than just four or five tidal periods. Apart from this suggestion for the collection of further data on internal tidal waves there are some additional data which are indispensable if a reasonably coherent picture of this important phenomenon is to be obtained. In order to explain these requirements it is necessary to consider briefly the theory of internal tidal waves.

II. Theory of Internal Tides

The internal wave motions which are possible in a stratified ocean have been studied by Fjeldstad (1933). He has also given a numerical integration method which permits the computations of the possible internal waves in an ocean with arbitrary density distribution. More recently Groen (1938) has studied internal waves in a fluid with a certain specified density distribution; namely, a rather rapid change in an intermediate layer and almost constant density in two very deep layers above and below the inhomogeneous layer. Here we shall confine ourselves to the discussion of a simple case, following Fjeldstad's work without reviewing his paper in detail.

If a rectangular Cartesian coordinate system is chosen with the x and y axes horizontal and the z axis vertically upward the velocity components may be denoted respectively as u, v, and w. Let $\ell = 2\omega \sin L$ be the Coriolis parameter, g the acceleration of gravity and $\bar{\rho}$ and $\bar{p}$ the density and the pressure, respectively. The fluid may originally be in a state of rest on which the internal wave motion is superimposed as a small perturbation. Then u, v, w are perturbation quantities. Further, let the density and pressure in the undisturbed state be distinguished by the subscript zero, ρ_0 and p_0 while the total density and pressure in the disturbed case are $\rho_0 + \rho$ and $p_0 + p$. The perturbation quantities, u, v, w, p, ρ are assumed to be so small that terms of higher order can be neglected. The fluid is incompressible, but the undisturbed density ρ_0 may be a function of z. Then the three equations of motion, the equation of continuity, and the equation expressing the constancy of the density of an individual parcel, are

$$
\begin{aligned}
&\frac{\partial u}{\partial t} - \ell v + \frac{1}{\rho_0}\frac{\partial p}{\partial x} = 0 \\
&\frac{\partial v}{\partial t} + \ell u + \frac{1}{\rho_0}\frac{\partial p}{\partial y} = 0 \\
&\frac{\partial w}{\partial t} + g\frac{\rho}{\rho_0} + \frac{1}{\rho_0}\frac{\partial p}{\partial z} = 0 \qquad (2.1) \\
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \\
&\frac{1}{\rho_0}\frac{\partial \rho}{\partial t} + \frac{1}{\rho_0}\frac{d\rho_0}{dz} w = 0
\end{aligned}
$$

The dependence of these variable on x, y and t may be expressed by

$$u, v, w, p, \rho \propto e^{\alpha y} e^{i(\sigma t - kx)} \tag{2.2}$$

This expression represents a wave progressing in the positive x direction with the speed $c = \sigma/k$, the period $2\pi/\sigma$ and the wave length $2\pi/k$. It assumes furthermore an exponential or periodic change in the y direction depending on whether the factor α is real or imaginary.

If all the other unknown functions are expressed by w one finds that

$$\frac{d}{dz}\left(\rho_0 \frac{dw}{dz}\right) + \lambda^2 g\left(\varphi - \frac{\sigma^2}{g}\right)\rho_0 w = 0 \tag{2.3}$$

where

$$\lambda^2 = \frac{k^2 - \alpha^2}{\sigma^2 - \ell^2} \tag{2.4}$$

$$\varphi = -\frac{1}{\rho_0}\frac{d\rho_0}{dz} \tag{2.5}$$

Further

$$\frac{p}{\rho_0} = -\frac{i}{\sigma\lambda^2}\frac{dw}{dz} \tag{2.6}$$

$$u = -\frac{i}{\sigma}\frac{k\sigma + \ell\alpha}{k^2 - \alpha^2}\frac{dw}{dz} \tag{2.7}$$

$$v = \frac{1}{\sigma} \frac{lk + \sigma\alpha}{k^2 - \alpha^2} \frac{dw}{dz} \tag{2.8}$$

Equations (2.7) and (2.8) show, when the real parts are taken, that

$$\frac{u^2}{A^2} + \frac{v^2}{B^2} = 1$$

where

$$A = \frac{1}{\sigma} \frac{k\sigma + l\alpha}{k^2 - \alpha^2} \left| \frac{dw}{dz} \right| e^{\alpha y},$$

$$B = \frac{1}{\sigma} \frac{lk + \sigma\alpha}{k^2 - \alpha^2} \left| \frac{dw}{dz} \right| e^{\alpha y}$$

Thus the hodograph of the tidal velocity is an ellipse. The ratio of the two axes

$$B/A = \frac{1 + \frac{\sigma}{l}\frac{\alpha}{k}}{\frac{\sigma}{l} + \frac{\alpha}{k}}$$

If α is complex or imaginary the foregoing representation of the hodograph would, of course, have to be modified.

The ratio of the two semi-axes can be obtained from observations. It depends on the period, the geographic latitude, which enters through the Coriolis parameter l, the wave length $2\pi/k$ and the factor α which determines the dependence on y. Hence, even if the period is given, as would be the case for tidal waves, only the ratio α/k can be determined. For more complete

information about the internal tides at least one of these two quantities, either the wave length or α , should be known.

Observations by three ships would permit one to find both α and the wave length, as well as the direction of travel of the internal wave, if it is progressive, and would thus greatly increase the information while at the same time allowing a more complete check on the theoretical deductions. Observations by more than one ship have been made by Fjeldstad during a study of internal tidal waves in a Norwegian fjord and by G. Neumann (1949) as part of an investigation of shorter-period oscillations. But from the open ocean such data have not been available at the time when this report was completed.

The differential equation (2.3) can be simplified, as shown in detail by Fjeldstad, since the small variation of the undisturbed density ρ_0 may be neglected in the first term on the left-hand side, so that

$$\frac{d^2w}{dz^2} + \lambda^2\left(g\varphi - \sigma^2\right)w = 0 \qquad (2.3a)$$

In order to make the problem determined it is necessary to introduce boundary conditions. At the same time the particular fluid model to be discussed here briefly will be specified. It may consist of a homogeneous layer, 1, extending from $z = 0$ to $z = h_1$ with a rigid lower boundary, and of an upper layer, 2, with variable density, extending from $z = h_1$ to $z = h = h_1 + h_2$, the free upper boundary. At the internal boundary the density is assumed to be continuous.

At the lower rigid boundary the vertical velocity must vanish,

$$w = 0 \quad \text{at} \quad z = 0$$

At the internal boundary the vertical velocity and the pressure must be continuous. Hence, with (2.6),

$$w_1 = w_2, \quad \frac{dw_1}{dz} = \frac{dw_2}{dz} \quad \text{at } z = h_1$$

At the free surface, z = h, a pressure condition has to be satisfied. However, we are only interested here in internal waves whose amplitude nearly vanishes at the free surface. Therefore, it is permissible to write for the boundary condition at the free surface

$$w = 0 \quad \text{at} \quad z = h$$

To consider a particularly simple case it will be assumed that in the upper layer with variable density ϕ is independent of z ,

$$\phi = const$$

Since

$$\rho = 1 + 10^{-3}\sigma_t$$

it follows that to a very close degree of approximation

$$\phi = -\frac{1}{\rho}\frac{d\rho}{dz} = -10^{-3}\frac{d\sigma_t}{dz}$$

Thus the assumption of constant ϕ implies a linear distribution of σ_t , the quantity which is actually determined in oceanographic surveys.

Since in the lower layer, 1, ϕ is assumed to be zero, while it is constant in the upper layer, 2, and since furthermore w_1 vanishes at z = 0, w_2 at z = h, one finds for the vertical velocity from (2.3a) that

$$w_1 = A_1 \sinh \lambda\sigma z \approx A_1 \lambda \sigma z$$

$$w_2 = A_2 \sin \eta (z-h)$$

where

$$\eta = +\lambda (g\phi - \sigma^2)^{1/2}$$

The simplified expression for w_1 follows because the depth of the fluid is much smaller than the wave length for the waves considered here.

At the internal boundary, $z = h_1$, the vertical velocity and its derivative with respect to z must be continuous. Hence,

$$A_1 \lambda \sigma h_1 = -A_2 \sin \eta h_2$$

and

$$\frac{\tan \eta h_2}{\eta h_2} = -\frac{h_1}{h_2} \qquad (2.9)$$

Further, the vertical displacement ξ is related to w by

$$\frac{\partial \xi}{\partial t} = w$$

Hence

$$\xi_1 = \frac{A_1}{i\sigma} \lambda \sigma z$$

and the amplitude of the vertical oscillation is at the internal boundary $z = h_1$,

$$C = \frac{A_1}{i\sigma} \lambda \sigma h_1$$

If this constant is used as the remaining integration constant

$$w_1 = i\sigma C \frac{z}{h_1} \qquad (2.10)$$

$$w_2 = i\sigma C \frac{\sin\eta(h-z)}{\sin\eta h_2} \qquad (2.11)$$

The relation between wave length and period is given by (2.9) which is a transcendental equation with an infinite number of roots. It can be solved in individual cases by approximation methods.

Of practical importance is the case that the lower layer is much thicker than the upper one. In this case a good approximation can be obtained in the following way. Since the right-hand side of (2.9) is quite large and negative

$$\eta h_2 = (2n+1)\frac{\pi}{2} + \varepsilon$$

where n is an integer and ε a small quantity. Hence, approximately,

$$\tan \eta h_2 = -\frac{1}{\varepsilon} = -\frac{1}{\eta h_2 - (2n+1)\frac{\pi}{2}}$$

If this expression is substituted in (2.9) it follows that

$$\eta h_2 = \frac{2n+1}{2}\pi + \frac{2}{(2n+1)\pi}\frac{h_2}{h_1} \qquad (2.12)$$

Since
$$\eta^2 = \frac{k^2 - \alpha^2}{\sigma^2 - l^2}(g\varphi - \sigma^2)$$

and since $g\varphi \gg \sigma^2$ it follows that the wave length

$$\frac{2\pi}{k} = \frac{4h_2}{2n+1}\sqrt{\frac{g\varphi}{\sigma^2 - l^2}}\left[1 - \frac{4}{(2n+1)^2\pi^2}\frac{h_2}{h_1}\right] \qquad (2.13)$$

As a numerical example let

$$\phi = 2.10^{-7}\ \mathrm{cm}^{-1}$$

$h_1 = 2700$ m, $h_2 = 140$ m,

Latitude 17° $\sigma = 2\pi/12.4$ hours

The figures correspond approximately to those at Meteor Station 385 (see below), except for the surface layer where the density increase is more rapid than is assumed here.

For lack of any better information about the distribution of the internal tidal waves perpendicular to the direction of propagation, it may be assumed that $\alpha = 0$. Then for various values of n the following wave lengths would have to exist in the case of internal waves with the period of the lunar semidiurnal tide

n	0	1	2	3
wave length	61	20	12	8.6 km

From the foregoing discussion it is evident again as from the discussion of the current vector that for a complete check of the theory simultaneous observations from more than one ship are required, in order to determine the wave length and the distribution of the oscillation in lateral direction.

If such internal oscillations occur they must show themselves also in periodic changes of conservative properties. If s denotes such a conservative property,

$$\frac{ds}{dt} = \frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = 0$$

Hence local variations, $\frac{\partial s}{\partial t}$, of the property can in general be due either to horizontal or to vertical motions. If the horizontal gradient of the property is small the terms depending on horizontal motion can be neglected. Further, it can be assumed that

$$s = \bar{s}(z) + s^*(x, y, z, t)$$

where the first term on the right-hand side stands for the mean value of s at each level z on which the small variation s^* due to the internal wave is superimposed. Then

$$\frac{\partial s^*}{\partial t} = -w \frac{\partial \bar{s}}{\partial z}$$

or if ξ denotes the vertical displacement of the particle

$$s^* = -\xi \frac{\partial \bar{s}}{\partial z}$$

Thus, ξ can be determined from the variation of s and the vertical gradient of the mean value of s.

In the following analyses we shall confine ourselves to the temperature variations, as explained previously. In the case of the Atlantis data time cross sections of the isotherms were already available so that it was possible to analyze the heights of given isotherms rather than the temperatures at given heights. Therefore, the same method has been used in the analysis of the Meteor data, although temperatures at given heights were studied in the case of the Altair data in order to save time and labor. It need hardly be mentioned that the temperature is not a strictly conservative property of sea water; but for the purposes of the present

investigation the errors arising out of this assumption are certainly not greater than those due to other causes.

III. Statistical Procedure

The numerical procedure for studying the various series of observations and the occurrence or non-occurrence of internal oscillations has been described briefly in the preceding report (Haurwitz, 1952). Since additional comments on the method are to be made here it will be best to review the whole procedure.

If a period of a certain length is suspected in a set of data it can be subjected to a harmonic analysis according to well-known methods. As a result one will obtain an expression, or a sum of expressions, of the form

$$a_n \cos nt + b_n \sin nt \tag{3.1}$$

Here t is the independent variable, for instance the time, expressed in angular measure, n an integer representing the frequency, and a_n and b_n the harmonic coefficients. An alternate form of the foregoing expression is

$$A_n \cos(nt - \alpha_n) \tag{3.2}$$

where

$$A_n = \sqrt{a_n^2 + b_n^2} \tag{3.3}$$

$$\tan \alpha_n = \frac{b_n}{a_n} \tag{3.4}$$

Since it is always possible to represent a set of data by a series of terms of the form (3.1) or its alternate (3.2) the harmonic analysis does not prove anything about the reality of the suspected periods. Even a set of random numbers can be represented by a harmonic series.

In many cases the existence of certain periods in the observation material will, of course, be highly likely because of the existence of a generating force. In the present case, for instance, it is known that the ocean is subjected to the tidal action of the sun and the moon. Therefore, it can be expected that the oceanographic parameters show always variations with the tidal periods. But these variations may be so minute that they are completely overshadowed by random variations, and that the results of a harmonic analysis represent merely the effects of such random variations. Hence a statistical test is necessary to decide whether the computed harmonic coefficients represent a real period or only random fluctuations.

Such a test for reality is provided by the expectancy method. The expectance is a measure for the amplitude A_n of a harmonic component to be expected in random data. The theory of probability shows what the probability is that the actual amplitude exceeds the expectance by a certain amount. Evidently if this probability is very small it is unlikely that the computed amplitude is due to random data, or in other words, it is likely that the period exists in the analyzed data.

The expectance is given by the following expression

$$E^* = 2\mu/\sqrt{n}$$

where μ is the standard deviation of the individual n numbers (for instance, Chapman-Bartels, 1940 or Stumpff, 1940). E^* would represent the mean amplitude in such a set of random data. It can be shown that the probability that any amplitude is k times the value of E^* is given by

$$p = e^{-k^2}$$

The application of the expectance criterion is, however, restricted to those cases where the individual members of the set of numbers are independent from each other. This is true for random numbers, but not for the majority of geophysical data. In the cases to be considered here, for instance, the subsequent heights of the isothermal surfaces will have a certain autocorrelation since evidently large positive deviations from the mean value are more likely to be followed by other large positive deviations from the mean than by large negative ones.

It is possible to avoid or at least to reduce greatly the effects of the autocorrelation by introducing instead of the expectance a somewhat different expression, the expectancy. In order to determine this quantity let the whole analysis interval $t_1 - t_0$ be an integral multiple of the period T whose reality is to be determined,

$$t_1 - t_0 = S \cdot T$$

This can always be arranged by suitable choice of the end points t_1, t_0. The total interval is now subdivided into equal parts each of which is either equal to the period T or to a multiple of T. The harmonic coefficients $a_n^{(i)}$ and $b_n^{(i)}$ are then determined separately for each partial interval of which there may be r, say, where $r \leqq S$. Let

$$A_n^{(i)^2} = a_n^{(i)^2} + b_n^{(i)^2} ,$$

the amplitude for each partial interval. The determination of the different $A_n^{(i)}$ is based on a number of points which with the exception of the last points of one partial interval and the first points of the next partial interval are pretty far apart so that the effects of any autocorrelation in the data is greatly reduced. Therefore, it can be assumed that the $A_n^{(i)}$ determined for the different partial intervals would in the absence of the suspected periodicity behave like random numbers and scatter around a mean value which may be assumed zero a priori. The harmonic coefficients $a_n^{(i)}$ and $b_n^{(i)}$ can be considered as the components of a vector which may be plotted in a coordinate system with a_n and b_n as the horizontal and vertical axes, respectively. This representation is known as the harmonic dial, which will be discussed later. When the various $a_n^{(i)}$, $b_n^{(i)}$ are plotted on such a harmonic dial the end points of the vectors will form a point cloud which should in the case of complete randomness of the data be distributed around the origin with circular symmetry.

The quadratic mean value of the lengths of the vectors which represent the harmonic coefficients of the individual partial

intervals is called the expectancy of these individual vectors,

$$e^2 = \sum_{i=1}^{r} A_n^{(i)^2}/r$$

Stumpff (1940) has shown that it is convenient to introduce the expectancy E for the distance of the center of gravity of the cloud of r points $a_n^{(i)}$, $b_n^{(i)}$ from origin,

$$E^2 = \frac{e^2}{r} = \frac{1}{r^2} \sum_{i=1}^{r} A_n^{(i)^2} \qquad (3.5)$$

The distance of the center of gravity of the point cloud from the origin is given by

$$\bar{A}_n^2 = \frac{1}{r^2}\left(\sum_{i=1}^{r} a_n^{(i)}\right)^2 + \frac{1}{r^2}\left(\sum_{i=1}^{r} b_n^{(i)}\right)^2 \qquad (3.6)$$

and the probability that

$$\bar{A}_n = k\,E \qquad (3.7)$$

is

$$p = e^{-k^2} \qquad (3.8)$$

for random data. Hence, if the probability p is small it is likely that the data are not randomly distributed, but that the suspected period is real.

Since the derivations which lead to the foregoing formulae are based on statistical theory they presuppose that the number of intervals, r, on which the test is based is not too small. This can be seen directly if the following case is considered.

Suppose that the $a_n^{(i)}$ and the $b_n^{(i)}$ for all partial intervals are the same and that the amplitude for each partial interval is A_n. In this case $p = e^{-r}$. If r = 2, for instance, the probability is one in 7 that any amplitudes, even though they are the same for both partial intervals, are due to random data. The expectancy test is, of course, not applicable to so few data, and the example merely illustrates this point. We shall, however, have to apply it later to data where r = 4 and where therefore the lowest probability is one in 55 that the amplitudes found by the harmonic analysis are due to random data. Even in this case the application of the test can only be justified because longer series are not available.

Another point to be discussed here concerns the exact length of the period whose reality is to be tested. If a sufficiently long series of observations is available the choice of a period slightly different from the exact one would manifest itself in a systematic change of the phase constant as each consecutive period interval is analyzed. But in a short series comprising only a few periods this variation of the phase constant would be completely obscured by the irregular changes due to superimposed random fluctuations in the data.

In the particular problem studied here this difficulty may arise because of the slight difference between the period $T_e = 12.42$ hours of the principal lunar tide and the period $T = 12$ hours of the principal solar tide. While the former is as a rule the dominant one this is not always true, and it can therefore not definitely be ascertained from a short series of observations which of the two periods is the correct one, even if the expectancy test is positive. In order to see how the choice of a period slightly different from the correct one, say T_e, affects the expectancy test assume that a time series is given by the simple expression

$$f(t) = A \cos \nu t$$

where

$$\nu = 12 \text{ hrs}/T_e$$

and t is the mean solar time in angular measure. Let the partial-analysis interval be T = 12 hours. Then

$$\nu = 1 - 0.034 = 1 - \varepsilon$$

The harmonic coefficients for the period T computed from the (k + 1) th analysis interval are

$$a_1^k = \frac{1}{\pi}\int_{2\pi k}^{2\pi(k+1)} f(t)\cos t\,dt = \frac{\sin \pi\varepsilon}{\pi\varepsilon}\,\frac{2(1-\varepsilon)}{2-\varepsilon}\,a\cos[\pi\varepsilon(2k+1)]$$

$$b_1^k = \frac{1}{\pi}\int_{2\pi k}^{2\pi(k+1)} f(t)\sin t\,dt = \frac{\sin \pi\varepsilon}{\pi\varepsilon}\,\frac{2}{2-\varepsilon}\,a\sin[\pi\varepsilon(2k+1)]$$

Because ε is small these expressions may be written approximately as follows

$$a_1^k = \left(1 - \frac{\varepsilon}{2}\right) a \cos\left[\pi\varepsilon(2k+1)\right]$$

$$b_1^k = \left(1 + \frac{\varepsilon}{2}\right) a \sin\left[\pi\varepsilon(2k+1)\right]$$

These two expressions show that for the fictitious period T the amplitude as determined from each partial analysis interval remains nearly constant and nearly equal to A while the phase constant changes by $2\pi\varepsilon$ from one interval to the next.

If the coefficients a_1^k and b_1^k are computed for the first four intervals of length T one finds that

$$p = 0.022$$

while the lowest value for the probability of randomness for r = 4 is 0.018. Evidently, the difference between these two values is too small to permit any decision about the correctness of the assumed period length.

But even with a longer series of observations the expectancy test cannot be used for a decision about the correct period length. If in the foregoing example 11 consecutive periods of the length T were used one would find that p = 0.0011 while the theoretical minimum value is 0.000017. The difference is here considerably larger. However, since in actual cases the observations would also be subjected to random effects a value of p larger than the theoretical minimum can hardly be attributed to the wrong choice

of period. On the other hand, with such a longer series the individual harmonic coefficients would show, despite superimposed random effects, a systematic change of the phase constant if the assumed period deviates from the actual one present in the observation material.

The foregoing remarks require some modifications if the partial analysis intervals consist of more than one period. But since the series analyzed here are not long enough to permit such a combination of the data this case need not be considered here.

IV. Analysis of the Data

A summary of the stations which were investigated is given in Table I. Station 3091 of Atlantis Cruise 77 which was discussed

Table I

LIST OF STATIONS

Designation	Location	Duration	Reference
Atlantis Cruise 77	34.1°N 65.9°W	16-29 June 1938	Haurwitz, 1952
Meteor Station 385	16.8°N 46.3°W	12-14 Feb. 1938	Schubert, 1944
Meteor Station 438	30.0°N 43.8°W	26-28 Apr. 1938	Schubert, 1944
Altair	44.5°N 34.0°W	16-20 June 1938	Defant, 1940a

in the earlier report has been included for completeness. The average vertical density distributions as obtained from observations made at these stations are shown in Fig. 1. In the case of the Atlantis and the Meteor observations one series of measurements was extended to much greater depths. The results of these

deep soundings are indicated by individual points not connected by a continuous curve. In the upper layers where mean values from the serial observations are also available the observations of the single deep sounding agree very well with the means of the serial observations. The average depth during each station is indicated by the horizontal line at the bottom of each diagram. The number gives the depth in meters. In the case of the Altair which anchored over a seamount the greatest and smallest depths are shown.

The vertical density distribution during the Atlantis station shows a very rapid density increase downward in the uppermost 100 meters, followed by a more homogeneous layer. Between 500 and 1200 m the density gradient is again slightly larger. Similarly, the Meteor Station 385 and the Altair Station show a surface layer of large density gradient followed by a more homogeneous layer. At Meteor Station 385 the density at the sea surface is even slightly higher than in the shallow homogeneous layer right below it. At Meteor Station 438 the density gradient in the highest layers is much less strong than at the other stations and considerably more uniform through the first 1000 m. The temperature distribution resembles the distribution of the density very closely in all cases.

e. Review of Atlantis Data

In order to recapitulate briefly the results of the study of the Atlantis data and to illustrate the method of presentation of the results of the present investigation, Fig. 2 shows the harmonic

dial for the 15°C isotherm. The average depth of this isotherm was 630 m. The analysis of the total time interval of observations which composed 21 lunar semidiurnal tidal periods was made for seven groups, each consisting of 3 consecutive tidal periods. The points denoted by A, B, ..., G represent the results of the individual analyses. The mean of all 21 periods is represented by the cross connected by a straight line to the origin of the coordinate system[1]. This mean amplitude A_m is only 1.23 times larger than its expectancy. Hence the probability that it is due to random data is 0.22, far too large to regard the computed amplitude for the lunar tidal period in the data as real. Instead of showing the expectancy of A_m in Fig. 2 circles of equal probability have been drawn with the origin as the center. The radius of each of these circles indicates the probability that a value of A_m of the same magnitude is due to random data.

The results of the analyses of the individual 7 groups, represented by the points A through G, scatter widely. In order to have a measure of this scatter the deviations $\Delta a_n^{(i)}$ and $\Delta b_n^{(i)}$ of the individual $a_n^{(i)}$ and $b_n^{(i)}$ from the mean have been formed. Then the quantity

$$0.67\sqrt{\frac{1}{r}\left[\sum(\Delta a_m^{(i)})^2 + \sum(\Delta b_m^{(i)})^2\right]}$$

has been computed. The resulting number may be considered as the

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[1] In Table I of the previous report the mean value of the b_2 for the 15° isotherm should be -1.63 m, rather than -1.90 m. Consequently in Table II of that report A_m = 4.05, k = 1.23, and p = 0.22, a change of no significance.

probable error of A_m. Its magnitude, 5.2 m, is indicated by the circle around the end point of A_m as the center. The fact that the probable error is larger than the value of A_m shows again that no tidal period is present in the Atlantis data.

The harmonic dial shows also, on the margin of the figure, for each position of the harmonic coefficient that time, in local lunar hours, at which the maximum would occur. Because of our choice of the expression for the harmonic representation, (3.2), the hours increase in the counterclockwise direction, opposite to the conventional clock faces.

b. Meteor Station 385

The location of this station is given in Table I. The serial observations extend only through the uppermost 150 m. Therefore the analysis has to be confined to this layer. As already pointed out by von Schubert (1944) an inspection of the figures showing the heights of isothermal and isohaline surfaces as functions of time (his Figures 7 and 8) shows fairly clearly a semidiurnal tidal period. Hence it could be surmised that the expectancy test should in the present case give a positive result as to the reality of the semidiurnal lunar tide. In order to perform the analysis time cross sections of the isothermal surfaces 24.5°C, 23°C, 22°C, 21°C, and 20°C were drawn on the basis of the published soundings, and values of the heights of these surfaces were read off for each lunar hour beginning with 0100 local lunar time, corresponding to 12 February 1938, 11:15 45°W meridian time. Altogether, hourly soundings were taken for 60 solar hours, but because of the greater

length of the lunar hour only four semidiurnal lunar tidal periods were available. The harmonic analysis was performed separately for each individual tidal period and the results are listed in Table II. The table also gives, in the lowest line, the average depth of the various isothermal surfaces during the observation period.

From these harmonic coefficients the mean amplitude A_m, its expectancy E, the phase angle α_m, the probability p, that A_m is due to random data and the probable error of A_m, P.E. (A_m) have been computed. They are given in Table III. The main conclusion to be drawn from this table is that it is not very probable that the amplitudes of the semidiurnal oscillation as computed are due to random data, in other words that the existance of semidiurnal internal tide is very probable. The value of the probability p that the amplitude A_m is due to random data is between 0.02 and 0.03 for the different isothermal surfaces. This probability for randomness is not very small, but it should be remembered that with only four groups of data the lowest probability which the expectancy test could give is $e^{-4} = 0.018$.

Table III

EXPECTANCY TEST FOR THE LUNAR SEMIDIURNAL TIDE

METEOR STATION 385

ISOTHERM	24.5°C	23°C	22°C	21°C	20°C
A_m	11.2 m	13.5	14.4	13.9	11.8
α_m	242°	249°	250°	251°	257°
E	5.7 m	6.9	7.4	7.5	6.3
p	0.02	0.02	0.02	0.03	0.03
P.E.(A_m)	1.5 m	1.5	2.2	3.6	3.1

Table II

HARMONIC ANALYSIS FOR THE LUNAR SEMIDIURNAL TIDE

METEOR STATION 385

12-14 February 1938, 16.8°N 46.3°W

ISOTHERM	24.5°C		23°C		22°C		21°C		20°C	
group	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2
A	-6.9m	-10.2m	-6.8m	-11.5m	-4.6m	-13.4m	-2.6m	-11.9m	1.2m	-11.3m
B	-6.3	-12.4	-6.2	-13.8	-6.2	-10.0	-5.6	- 7.7	-5.0	- 5.7
C	-3.2	- 8.4	-3.8	-10.5	-6.0	-12.3	-6.4	-11.6	-3.0	-12.1
D	-4.2	- 8.5	-2.6	-14.7	-3.0	-18.4	-3.2	-21.7	-4.0	-16.8
Mean	-5.2	- 9.9	-4.8	-12.6	-5.0	-13.5	-4.4	-13.2	-2.7	-11.5
Mean depth	78m		95m		106m		116m		129m	

For the 23°C isotherm a harmonic dial similar to Fig. 2 for the Atlantis data is shown in Fig. 3. A comparison of the two figures shows clearly how much more probable the existance of the semidiurnal lunar tide is in the data from the Meteor Station 385 than in the Atlantis data.

At smaller depths than about 80 m, the mean depth of the 24.5°C isotherm, tidal oscillations could not be observed, as has been pointed out already by von Schubert (1944). Even if tidal oscillations were in existence here it would be very difficult to observe them because the vertical gradients of temperature and salinity are very small.

As Table III shows the amplitude increases downwards and reaches a maximum for the 22°C isotherm while the phase angle increases steadily, although somewhat irregularly. But the differences are not larger than the probable errors. Hence, it must be concluded that no significant variations with depth of the internal tidal oscillation can be found. Since the layer from which data are available comprises only 50 m this is not surprising.

As pointed out in section 3 it is not possible to decide on the basis of the short series of available data whether the period length is really equal to that of the principal lunar tide. In order to illustrate this difficulty on actual data a harmonic analysis was performed with the principal solar tidal period as the analysis interval. For this analysis not the height of an isothermal surface was chosen, but the temperature variation at 120 m depth which could be taken directly from von Schubert's (1944) tables.

Since the whole series comprises 60 hours five individual periods are available for the expectancy test. The probability that the mean amplitude at this level, 0.97°C is due to random fluctuations is 0.02. This figure is slightly smaller than the probability, 0.03 that the lunar-tidal harmonic coefficients of the 21°C and 20°C isotherms are due to random fluctuations. But because of the short length of the series of observations the slight difference between these probabilities has no significance.

In order to demonstrate that, on the other hand, the expectancy test shows the nonexistence of periods whose lengths differ substantially from the real ones the temperature variations at 120 m depths were also analyzed for a period of eight hours. The expectancy test showed that the resulting mean amplitude of 0.14°C would occur in random data with the probability 0.79. Hence such a period does not exist in the data.

c. Meteor Station 438

The exact location of this station which is about 13° north of Meteor Station 385 is shown in Table I. As Fig. 1 shows there is no layer of very rapid density increase near the surface. But a pronounced density gradient exists throughout the highest 800 m to which depth the regular serial observations were made. As at the other Meteor Station observations were made at hourly intervals, from 26 April 1938, 10:12 45°W meridian time to 28 April 1938, 20:58 45°W meridian time, a period of almost 60 hours. Thus again four consecutive lunar semidiurnal periods could be analyzed. As zero for t 1400 local lunar time on 26 April was chosen which

corresponds to 10:46 45°w meridian time. The temperature data published by von Schubert (1944) were again used to construct a time cross section of the height of various isothermal surfaces, viz., 18.5°, 18°C, 17.5°C, 17°C, etc., to 10°C from which the height of each surface for every lunar hour could be read. Even a casual inspection of this time cross section showed that internal oscillations of tidal character if present at all are much less well developed than at Meteor Station 385.

Table IV shows the harmonic coefficients for the four partial intervals. The results of the expectancy test are summarized in Table V whose arrangement is similar to Table III, but probable errors for the A_m and the mean phase angles have not been computed. Except for the 16°C and 14°C isotherms the probability is always greater than one half that the amplitudes found by harmonic analysis are due to random fluctuations. Even for the 16°C and 14°C isotherms the probability of randomness is not small enough to regard the existence of a semidiurnal oscillation as established.

Table V

EXPECTANCY TEST FOR THE LUNAR SEMIDIURNAL TIDE

METEOR STATION 438

ISOTHERM	18.5°C	17.5°C	16°C	14°C	12°C	10°C
A_m	3.1 m	1.0	5.0	1.9	1.9	2.4
E	4.0 m	1.5	3.3	1.4	2.9	3.3
P	0.55	0.63	0.11	0.16	0.65	0.58

Table IV

HARMONIC ANALYSIS FOR THE SEMIDIURNAL TIDE

METEOR STATION 438

26-28 April 1938, 30.0°N 43.8°W

ISOTHERM	18.5°C		17.5°C		16°C		14°C		12°C		10°C	
Group	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2
A	-1.5m	1.7m	-0.4m	0.1m	5.9m	0.6m	1.2m	3.3m	-2.2m	1.4m	-5.5m	0.5m
B	8.1	7.0	-2.1	2.1	-1.6	8.5	-0.6	0.2	2.2	-8.6	-6.9	-7.0
C	-3.0	8.9	1.8	-2.2	0.4	7.0	2.8	1.8	5.8	0.5	4.8	3.5
D	2.4	-6.8	1.8	-3.9	3.0	2.7	-1.8	2.3	-4.2	-0.8	-2.1	2.3
Mean	1.5	2.7	0.3	-1.0	1.9	4.6	0.4	1.9	0.4	-1.9	-2.4	-0.2
Mean depth	61m		220m		401m		533m		652m		778m	

It may be pointed out that the mean vertical temperature gradient has the following values:

Depth	0 - 100 m	100 - 300 m	300 - 400 m	400 - 800 m
°C/100m	.91	.54	1.03	1.56

Hence the local temperature changes from which the height changes of the isothermal surfaces have been computed are very small; even the amplitude of the 16°C isotherm, 5 m, corresponds only to about 0.07°C. The appearance of greater randomness of the higher isotherms, 18.5°C and 17.5°C, might therefore be attributed to the still smaller lapse rates in these layers. However, if this reasoning were correct the 12°C and 10°C isotherms should also show a lower probability of randomness than they actually do since they are in the layer between 400 and 800 m where the temperature decreases even more rapidly than between 300 and 400 m. It is, of course, possible that the existence of an internal tidal oscillation was confined to the vicinity of the 16°C isotherm because of its position near a discontinuity of the temperature lapse rate. But because of the rather large value of the probability p that the amplitudes are due to random data rather than an internal tide it does not appear necessary to consider this possibility here in greater detail.

d. Altair Station (1938)

The third series of observations was obtained by the Altair during the period from 16 to 20 June 1938 (Defant, 1940a). Hourly soundings were made for a period of 87 hours so that 7 complete

lunar semidiurnal tidal periods were available. The hourly soundings were made to a depth of 800 m. Instead of determining and analyzing the depths of various isothermal surfaces as in the case of the Meteor data the temperature changes at given levels will here be used directly following the procedure used by Defant (1940b). Further, instead of analyzing separately the temperature data at certain levels the temperature of the following levels have been added and their sums analyzed:

I	25 m,	50 m,	75 m	
II	100 m,	150 m,	200 m	
III	300 m,	400 m,	600 m,	800 m

This combination of data has the advantage that it smooths out some of the irregularities in the temperature variations at the individual levels. As pointed out by Defant the indicated selection of height combinations follows the actual stratification of the ocean at the Altair station into a surface layer down to 100 m with a sharp discontinuity at the surface, a middle layer of normal density increase to about 350 m and a lower layer with normal density increase separated from the middle layer by a weaker discontinuity layer.

For the analysis of the Altair data according to the lunar semidiurnal tidal period lunar time for Greenwich rather than for the local meridian was used. The origin of the time axis was 09:00 lunar Greenwich time corresponding to 17 June 1938, 00:02 GMT.

The results of the analysis for the three layers are summarized in Table VI. The individual and the mean temperature amplitudes

Table VI

HARMONIC ANALYSIS FOR THE LUNAR SEMIDIURNAL TIDE

ALTAIR STATION

16-20 June 1938 44.5°N 34.0°W

LAYER	I		II		III	
Group	a_2*	b_2*	a_2*	b_2*	a_2*	b_2*
A	0.34	-0.35	0.15	-0.19	0.34	-0.20
B	-0.16	-0.18	0.16	-0.06	0.02	-0.09
C	-0.22	-0.06	-0.02	-0.10	0.02	-0.01
D	0.30	-0.40	0.20	-0.02	0.10	0.14
E	-0.08	-0.20	0.08	0.06	-0.05	0.01
F	0.35	0.16	0.06	0.18	0.14	-0.11
G	0.28	0.64	0.10	-0.11	0.23	-0.24
Mean	0.116	-0.056	0.104	-0.034	0.114	-0.057
Mean Temp	14.85°C		12.79°C		10.77°C	

* The units of the harmonic coefficients for layers I and II are 0.33°C, for layer III 0.25°C.

are quite small, only of the order of a few hundredth degree Centigrade. In order to show the corresponding magnitude of the vertical displacements the mean vertical temperature gradients must be considered. These are as follows according to Defant (1940b):

Depth m	0	25	50	75	100	200	300	400	600	800
°C/100m	3.9	5.3	4.2	1.6	1.0	1.2	1.0	1.5	0.7	

For instance since the mean amplitude for layer II is about 0.036°C the corresponding amplitude of the vertical oscillation would be less than 4 meters. This value is, of course, computed on the assumption that the temperature variation is entirely due to vertical motion and not to horizontal advection. The latter possibility has been considered by Defant.

The expectancy test is summarized in Table VII.

Table VII

EXPECTANCY TEST FOR THE LUNAR SEMIDIURNAL TIDE

ALTAIR STATION, 1938

LAYER	I	II	III
A_m	0.043°C	0.036	0.032
E	0.054°C	0.022	0.021
p	0.53 (0.58)	0.06 (0.03)	0.10 (0.12)

The probability is very high that the amplitude for the semidiurnal lunar tide computed for the top layer is not real. The probabilities for the middle and lower layers are considerably smaller giving greater confidence in the reality of the internal tide at these levels, even though the values of p are not small enough to make the existence of the internal tide here a certainty. But despite the relatively high values of p for these two layers it is remarkable that especially in layer II where the temperature gradients are smallest the probability of randomness is also smallest. This fact suggests strongly that with a longer series

of observations the existence of the internal semidiurnal tide would have been brought out by the expectancy test. The numbers, p, in parentheses will be discussed later, on page 39.

From an inspection of the current measurments, and the temperature and salinity soundings Defant (1940b) surmised that a 17-hourly wave was superimposed on the semidiurnal lunar tidal oscillation. That an oscillation of 17 hours duration might exist is theoretically plausible since the period of the inertia motion at this latitude is 17 solar hours. Therefore the Altair observations have also been analyzed with respect to this period. Five complete inertia periods are contained in the time interval during which soundings were made. For the origin of the time axis the same time was chosen as in the case of the analysis for the lunar tidal period; viz., 17 June, 00:02 GMT. The harmonic coefficients are given in Table VIII

Table VIII

HARMONIC ANALYSIS FOR THE INERTIA PERIOD (17 Hours)

ALTAIR STATION, 1938

LAYER	I		II		III	
Group	a*	b*	a*	b*	a*	b*
A	0.11	-0.36	0.04	-0.21	0.22	-0.14
B	0.11	0.09	-0.05	-0.03	-0.01	0.09
C	0.27	-0.17	0.10	0.09	-0.14	0.20
D	-0.30	-0.17	0.13	-0.08	-0.07	-0.06
E	-0.45	0.12	-0.02	-0.03	0.01	0.14
Mean	-0.053	-0.10	0.037	-0.053	0.000	0.046

* The units of the harmonic coefficients for layers I and II are 0.33°C, for layer III 0.25°C.

The expectancy test is summarized in Table IX. Since the probability is large that amplitudes of the magnitudes found for the inertia period would be found in random data the reality of the 17 hour period is open to serious doubt.

Table IX

EXPECTANCY TEST FOR INERTIA PERIOD

ALTAIR STATION, 1938

LAYER	I	II	III
A_m	0.038°C	0.021	0.011
E	0.047°C	0.020	0.019
p	0.53	0.34	0.70

In order to test how well the superposition of the two periods, namely, the lunar semidiurnal and the inertia period, represents the observed data the mean temperature of layer II has been computed by harmonic synthesis for every lunar hour and compared with the observed mean temperature. The correlation coefficient between the 83 computed and observed values is only 0.54 indicating that the representation is not very satisfactory.

The existence of a 17-hour period in the Altair data appears thus unlikely. Nevertheless the possibility of the existence of two superimposed periods in the observation material raises the question as to the influence of the coexistence of two or more periods on the expectancy test.

V. The Superposition of Two Oscillations

Suppose that a function of time $\rho(t)$ is the sum of two oscillations of different periods, T_ℓ and T_1, so that

$$\rho(t) = A \cos \sigma_\ell t + B \sin \sigma_\ell t + \alpha \cos \sigma_1 t + \beta \sin \sigma_1 t \tag{4.1}$$

where $\sigma_\ell = 2\pi/T_\ell$, $\sigma_1 = 2\pi/T_1$. In order to perform the expectancy test with respect to the period T_ℓ one would determine the following coefficients by harmonic analysis

$$a = \frac{2}{T_\ell}\int_{kT}^{(k+1)T} \rho(t) \cos \frac{2\pi}{T_\ell} t \, d t, \qquad b = \frac{2}{T_\ell}\int_{kT}^{(k+1)T} \rho(t) \sin \frac{2\pi}{T_\ell} t \, d t$$

which become, on performing the integration

$$a = A + \frac{2}{\pi}\frac{T_\ell}{T_1}\frac{\sin \pi \frac{T_\ell}{T_1}}{\left(\frac{T_\ell}{T_1}\right)^2 - 1}\left\{\alpha \cos\left[(k+1/2)\ 2\pi\frac{T_\ell}{T_1}\right] + \beta \sin\left[(k+1/2)\ 2\pi\frac{T_\ell}{T_1}\right]\right\} \tag{4.2}$$

$$b = B + \frac{2}{\pi}\frac{\sin \pi \frac{T_\ell}{T_1}}{1 - \left(\frac{T_\ell}{T_1}\right)^2}\left\{\alpha \sin\left[(k+1/2)\ 2\pi\frac{T_\ell}{T_1}\right] - \beta \cos\left[(k+1/2) 2\pi\frac{T_\ell}{T_1}\right]\right\} \tag{4.3}$$

If the expectancy test were performed not by dividing the whole analysis interval into groups of the length of a single period,

but into groups which comprise an integral number of periods, the foregoing formulae would only have to be modified slightly.

The last formulae for a and b show that because of the presence of an additional oscillation the harmonic analysis will in general not give the correct values A and B for the coefficients of the period T_ℓ, but different values, the difference depending on the ratio of the periods T_ℓ and T_1 and on the magnitude of the coefficients α and β of the superimposed period. If the observation series is long enough and if the ratio T_ℓ to T_1 is an interger it is, of course, possible to eliminate the effect of a superimposed period when performing the expectancy test. For instance, if T_ℓ = 24 hours and the period to be studied T_1 = 12 hours, it is merely necessary to subdivide the whole set of data into groups of the length $2 \cdot r$ x 12 hours, in order to eliminate the effect of the 24-hourly wave on the expectancy test.

In the cases which have been studied here it might in general be suspected that a wave of approximately 24 hours period also exists because of the corresponding terms in the tidal potentials of the moon and of the sun. It is not possible to eliminate the possible effect of such a longer period here by the combination of a number of periods since the total lengths of the series are not large enough. However, if an appreciable 24-hourly oscillation were present it would show itself in systematic variations of the coefficients a_2 and b_2 for each individual time interval of 12 lunar hours. Since no such variations are apparent in the computed coefficients it must be concluded that the expectancy

tests have not been influenced by an oscillation with a period near 24 hours.

In the case of the Altair data with the suspected inertia period of 17 hours matters are more complicated because of the non-integral ratio of the two periods involved. It is possible to apply a correction, for instance to the harmonic coefficients of the lunar semidiurnal tide given in Table VI, on the basis of equations (4.2) and (4.3). The coefficients a and b may be identified with the coefficients as listed in Table VI. The correct coefficients would be A and B which can be computed if the coefficients α and β of the 17-hour oscillation are known. To make this correction these coefficients were identified with the mean values of the coefficients a and b of the 17-hour oscillation listed in Table VIII. Such a correction has been applied. The corrected values of a_2 and b_2 are not reproduced here. Only the changes in the probabilities of randomness due to the corrections are shown, namely by the figures in parentheses given in the last line of Table VII. In general the changes due to the corrections are slight. Only in the case of the middle layer II, the probability that the computed amplitudes are due to random fluctuations is halved a fact which lends further weight to our previous remarks about the reality of internal tide in this layer. It need hardly be pointed out that the whole correction method is very crude since the time variation of the data is only very inadequately represented by the superposition of a lunar semidiurnal and an inertia oscillation. The correction has been applied here mainly to anticipate a possible objection to the expectancy test.

VI. Conclusion

The preceding analyses show that internal tidal waves appeared with reasonable certainty only in one case, namely, at Meteor Station 385, while at the other stations the expectancy test gives either inconclusive results or shows that no such oscillations occurred.

It is apparent from the discussion presented here that only longer series of data lasting more than just 2 or 3 days permit a decision as to the reality of internal tidal waves, and it is one of the principal aims of this report to stress the need for such adequate observations. Furthermore, the value of any series of observations will be greatly enhanced if measurements are taken simultaneously at three observation points, as explained in Section II. Such synoptic observations would not only support each other, but at the same time give much needed information about the wave length and direction of travel of these internal waves and about their lateral extent.

The question whether these internal waves are due to horizontal or vertical motions or to a combination of both has only briefly been touched here since it is beyond the scope of this report. Also for a decision on this question observations from three points would be very helpful since they would give the necessary information about the horizontal gradients of the required oceanographic parameters. In addition, the observations should also include current observations which would show whether the horizontal advection together with the horizontal gradients could

alone account for the observed changes or not. Moreover, since the internal waves must produce current fluctuations data on these currents would further corroborate any tidal fluctuations observed in temperature of salinity.

Acknowledgements

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List of Figures

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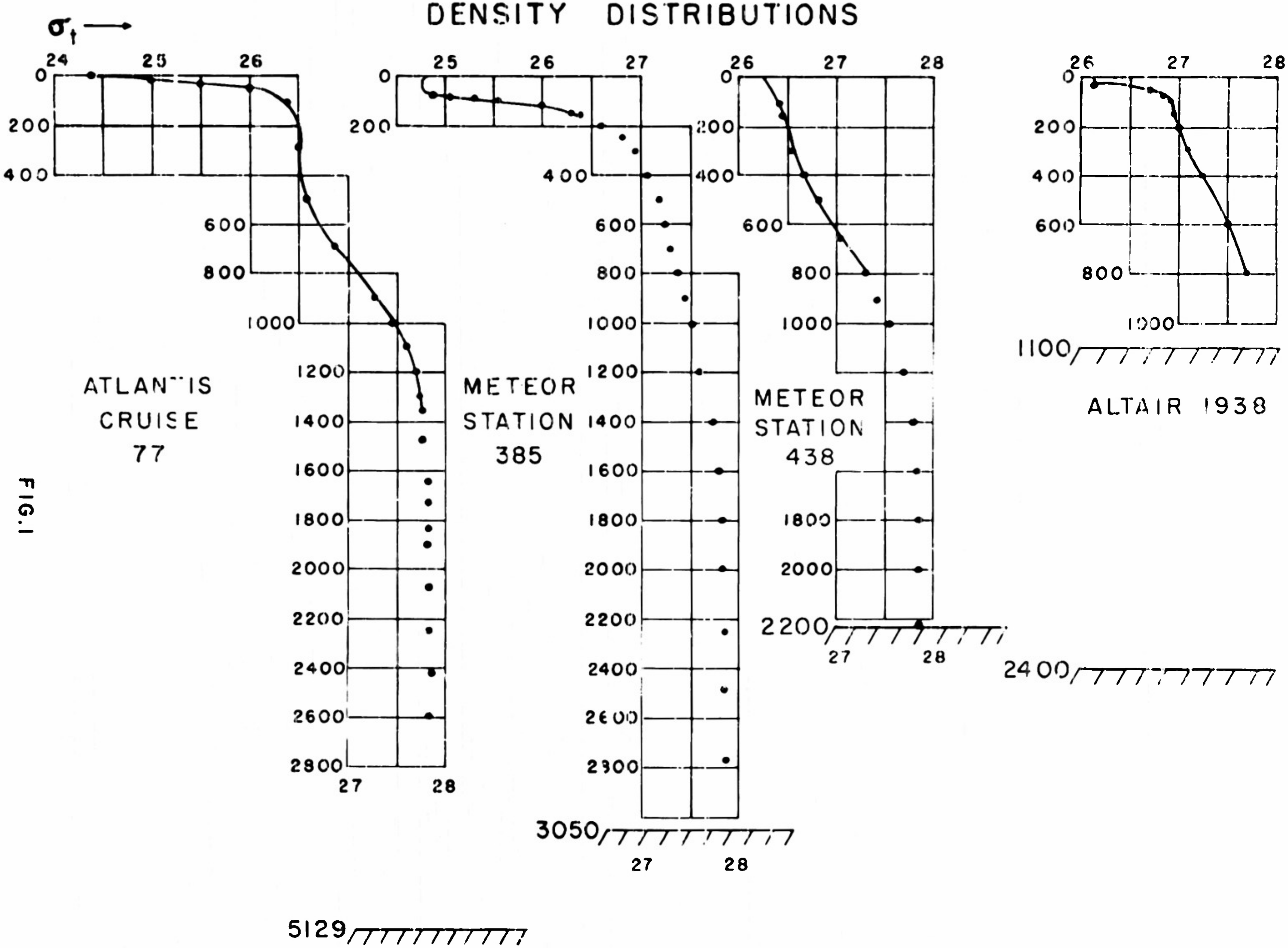

FIG.1

HARMONIC DIAL FOR THE LUNAR SEMI-DIURNAL OSCILLATION OF THE 15°C ISOTHERM

ATLANTIS CRUISE 77

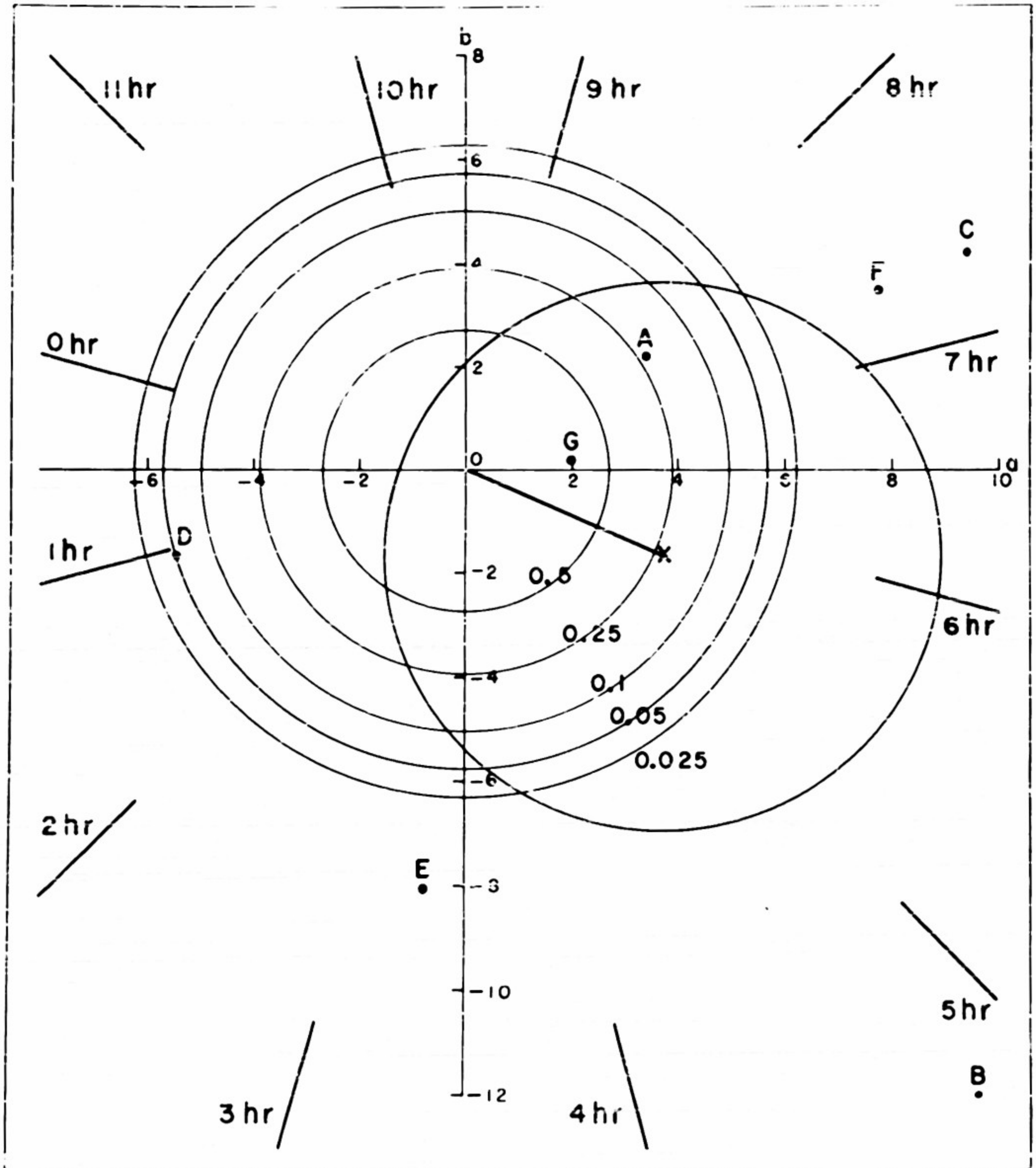

FIG. 2

HARMONIC DIAL FOR THE LUNAR SEMI-DIURNAL OSCILLATION OF THE 23°C ISOTHERM

METEOR STATION 385

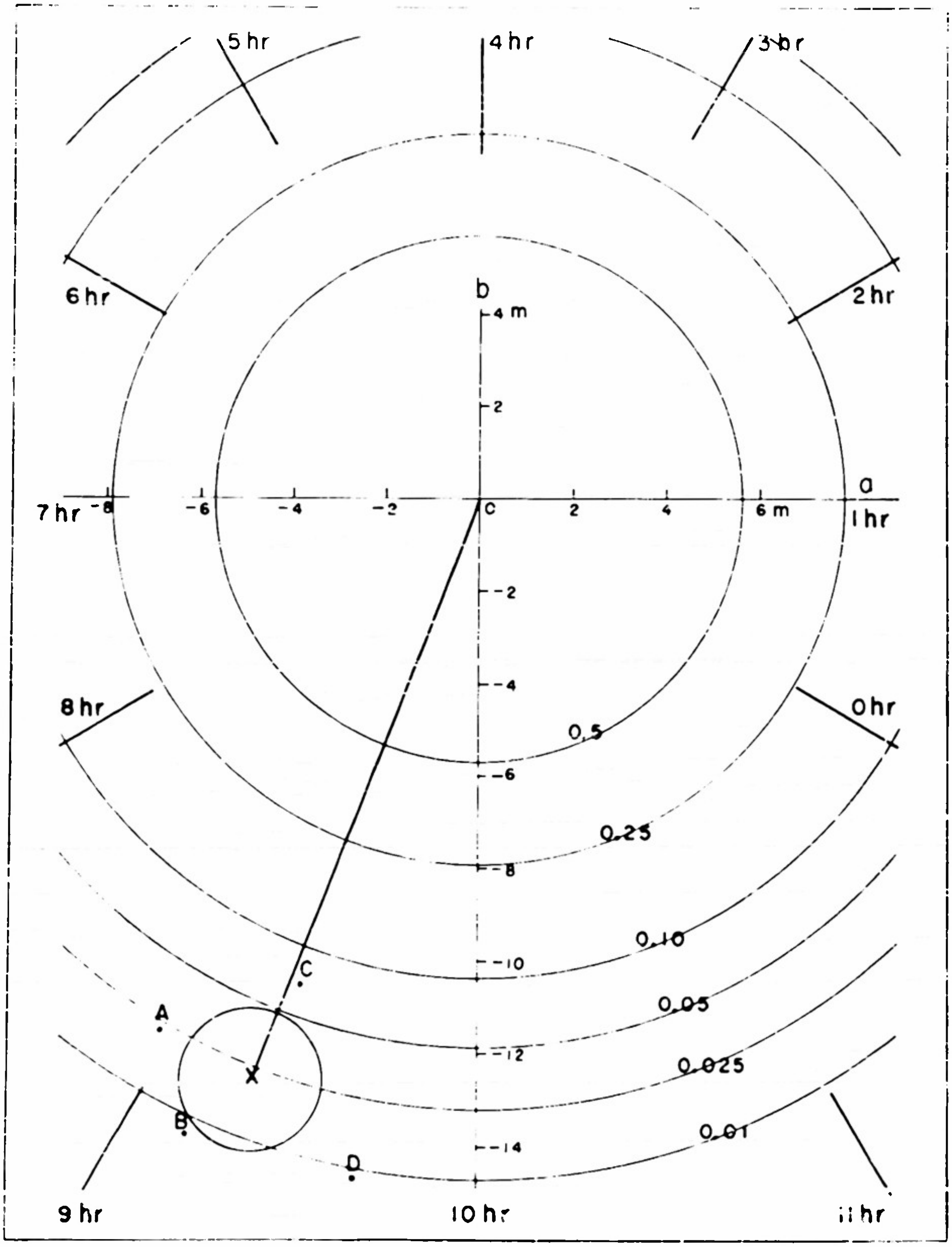

FIG. 3

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